

Research of 3D Irregular Fragment Reassembly Technique in Reverse Engineering

NIE Bo-lin, ZHANG Xu, CHE Xuan-lin

(School of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai, 201620)

Abstract:- in order to achieve some pieces of important research value of restructuring, this paper aiming at the shortcomings of the current existing algorithm proposed a new spatial 3 d irregular fragments stitching method. First Uses the method to establish point cloud based on kd - tree search space the topology relationship, for k neighborhood search quickly, further realize the boundary extraction of point cloud fragments; Secondly using Nurbs curve interpolation method to deal with boundary feature points, complete boundary fit at the same time the curve interpolation points of curvature and torsion; Finally based on the calculation of curvature and torsion, according to the longest common subsequence complete curve matching, and then complete the point cloud fragments of stitching. Through examples show that the proposed algorithm good robustness, high matching precision.

Keywords : - Reverse Engineering ;Curve fitting ;Curve matching ;The Longest Common uence (LCS) ; fragment reassembly

I. INTRODUCTION

So-called 3 d fragments split is some irregular space debris joining together into a complete model of the initial. In our country, ceramic relics buried in the ground, because of geographical or man-made reasons, the unearthed will be destroyed into large and small pieces. As a result of these pieces of cultural relics still has the very high archaeological value, therefore how to use modern technology to restore these cultural relics has important research significance.

Currently, the main space debris in the fracture restoration contour curve matching, as part of the curve matching problem. In recent years, many Chinese and foreign scholars on curve matching this sort of question to do a lot of research., the earliest Besl^[1] and proposed recently to 3 d space curve matching point iteration (iterative closest point, ICP) method, but as a result of this method requires an empty a subset of the curve is another empty asked curve, but in pieces joining together, the two can match the intersection, the space curve can be not as the empty set, rather than an outline is another subset of contour, so this method is difficult to directly used for contour curve matching of fragments merging. Ucoluk G^[2] the longest common subsequence algorithm is proposed, such as sampling points on the curve curvature and torsion as the characteristic, according to the similar feature points of similar matrix, using the dynamic programming algorithm to find the longest matching sequence, as the optimal matching, the algorithm deficiency is high time complexity, the ideal smooth curve matching effect is good, does not apply to any free curve shape. Wolfson^[3] to curve resampling in the first place, such as computing the curvature of the sample point, the curvature to characters value, and then according to the matching hash algorithm. Steven E^[4] according to the curvature of the contour curve feature points such as value, take the Pearson coefficient calculation according to the similarity of two curves. Shin H^[5] through statistics such as mural contour curve geometric features include information such as length, area, four crown, compare their similarity. Oxholm G^[6] computing the curvature of the contour curve of sampling points, such as burning rate and the color of character string, to take the longest common substring realize contour matching.

Ding Xianfeng^[7] on existing two-dimensional curve matching algorithm to carry on the induction and summary. Pan Rongjiang^[8] etc is put forward based on the longest common subsequence (LCS) fragments of the contour curve matching algorithm is used to match, according to the characteristics of point contour curve segment, and find out the LCS between two piecewise curve, the curve of the spell of overlapping tests results, it is concluded that the optimal matching. Lv Ke etc^[9] will outline curve segment, and put forward the measure curve segment similarity hash vector, using contour line segment matching algorithm based on Fourier transform, by comparing the two hash of the profile of the vector for the analysis of curve segment is similar. Jian wang^[10], etc according to the curvature of the sampling points get two curves on match point, through the alignment of the matching point Frenet frame curve matching. Shu-cheng zhou^[11] presents a using wavelet multiscale description of contour curve matching algorithm. Zhu Yanjuan etc^[12] are given based on the sampling point of curvature and torsion of the contour curve matching algorithm, according to total curvature to extract

the feature points, and the contour curve segment, according to the feature point on the piecewise curve curvature and torsion judgment piecewise curves is similar, and then according to the method of vector for further verification, the curve of the high similarity of 3 d transform at the same time, they are aligned, so as to realize the splice pieces.

II. FRAGMENT REASSEMBLY

2.1 boundary extraction and boundary fitting

2.1.1 boundary extraction

This article adopts the method of tree search based on kd - a point cloud space the topology relationship, k neighborhood search quickly. In the mining point P and the search for k neighboring points M_j ($j = 0, \dots, k - 1$) as a local reference data, using the least squares fitting out the tangent plane. Assume that $F(x, y, z) = a_1x + a_2y + a_3z + a_4 = 0$, for the micro tangent plane of $k + 1$ point, written in matrix form is:

$$\begin{bmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ \dots & \dots & \dots & \dots \\ x_k & y_k & z_k & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = 0 \quad (1)$$

$$A = \begin{bmatrix} x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ \dots & \dots & \dots & \dots \\ x_k & y_k & z_k & 1 \end{bmatrix}, a = [a_1, a_2, a_3, a_4]^T. \text{ So the } Aa = 0. \text{ Singular value}$$

decomposition of $A^T A$:

$$A = U \begin{bmatrix} \Delta & 0 \\ 0 & 0 \end{bmatrix} V^H \quad (2)$$

Type in the U and V as the unitary matrix, $\Delta = \text{diag}[\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_r}]$ and the singular value of matrix A is $\sqrt{\lambda_i}$ ($i=1,2,3,\dots,r$), which are characteristic values for $A^T A$. r is the number of singular values, and that number is eigenvalue of the matrix. $A^T A$ eigenvectors corresponding to the minimum eigenvalue is the least squares solution of $Aa = 0$. The micro tangent plane normal vector is $n = (a_1, a_2, a_3)$.

The sampling points and their neighborhood K corresponding to the projection plane microdissection to obtain the projected point P'_i and M'_i ($j = 0, 1, 2, \dots, k - 1$), the point set scattered into general distribution. Calculated projection coordinates, assuming space unorganized points coordinates $N_i(x_i, y_i, z_i)$ ($i = 0, 1, \dots, n$), which coordinates the micro-projection plane is cut (x'_i, y'_i, z'_i) , so there are:

$$a_1x'_i + a_2y'_i + a_3z'_i + a_4 = 0 \quad (3)$$

$$\frac{x'_i - x_i}{a_1} = \frac{y'_i - y_i}{a_2} = \frac{z'_i - z_i}{a_3} = k \quad (4)$$

In the equation,

$$k = \frac{a_1x_i + a_2y_i + a_3z_i}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Simultaneous equations (1) and (2) can be obtained projection point coordinates (x'_i, y'_i, z'_i) .

Construction $xP'x$ plane coordinate system, the micro-cut plane to P'_i origin to P'_i to the vector direction from the farthest point P'_i posed for the x -axis direction, perpendicular to the x -axis direction the y direction, the projection on the tangent plane of the micro-sample point, if its K neighbors projection points evenly distributed around the sample point, then the sampling point non-boundary characteristic points; if K neighborhood projection points around the sampling point unevenly distributed, it is considered that the sampling point for the boundary characteristic point [13]. In P'_i as a starting point for its K points in the neighborhood on the micro-projection tangent plane as the end point defined vectors $P'_iM'_j$, the three-dimensional coordinates of the projection point conversion to cut the micro plane, and the plane parameters microdissection of. Projection points x, y coordinates are in $P'_iM'_j$ vector corresponding to the projected length M'_jx and M'_jy of x -axis and y -axis, thereby obtaining projection point coordinates M'_j the parameterized (M'_jx, M'_jy) .

Since Scattered clouds in the form of discrete spatial distribution of vector projected point is generally composed of random distribution, in order to facilitate the calculation, the first projection vector is normalized:

$$P'_i M_j'' = \frac{P'_i M'_j}{|P'_i M_j''|} = \frac{|M'_{jX}, M'_{jY}|}{\sqrt{M'^2_{jX} + M'^2_{jY}}} = (X_i, Y_j) \quad (5)$$

In the equation, $j=0,1,2,\dots,k-1$. and $j \neq i$. For vector $P'_i M_j''$, giving it a force F_{ij} , the direction of the force vector in the same direction, the size of the force is equal to $|P'_i M_j''|$, then by calculating the K points in the neighborhood of sampling points for force $\sum F_{ij}$ in component A and B on X and Y axes to identify the boundaries of the feature point, component calculated by the following equation:

$$\left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} F_{ij} \right)_X = \left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} P'_i M_j'' \right)_X = \sum_{\substack{j=0 \\ j \neq i}}^{K-1} X_j \quad (6)$$

$$\left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} F_{ij} \right)_Y = \left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} P'_i M_j'' \right)_Y = \sum_{\substack{j=0 \\ j \neq i}}^{K-1} Y_j \quad (7)$$

First, set a threshold σ , when

$$\frac{\left| \left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} F_{ij} \right)_X \right|}{K} > \sigma$$

or

$$\frac{\left| \left(\sum_{\substack{j=0 \\ j \neq i}}^{K-1} F_{ij} \right)_Y \right|}{K} > \sigma$$

it can be determined that the sample point as a boundary point.

2.1.2 Boundary Fitting

Sort boundary point

Boundary point of the feature extraction algorithm was presented random distribution, in order to obtain a continuous boundary line, the need for border feature points are sorted to facilitate subsequent processing point cloud data. Using the nearest point search algorithm to sort the boundary points,

The steps are as follows:

- (1) Take any centralized point P_s at boundary points extracted as a starting point to find the point of focus from the nearest point P_e and P_s as the growth end;
- (2) $P_s (P_e)$ end, to find the distance $P_s (P_e)$ of the nearest point P , P to calculate P_s and P_e distance d_s and d_e , if $d_s (d_e) \leq d_e (d_s)$, will be inserted into the $P P_s$ before (P_e) , and the point P as a new starting point (end point); otherwise, to the other end of the growth;
- (3) all point set points to determine whether the completion is the end of the sort; otherwise, go to step (2).

Boundary characteristic points sorted though a simple straight line connecting faster, but the boundary line was simply C0 continuous (position continuous), not smooth enough, is not conducive to the subsequent processing, and for the most complex parts and molds, which boundary lines are generally curved, so this paper the method of NURBS curve interpolation boundary feature point processing.

1) knot vector solution

In order to make a k -th B-spline curve by setting values to the point $P_i (i = 0,1, \dots, n)$, its inverse process is generally the first end of the curve to make the two endpoints and data points corresponding to the beginning and end points consistent, each node data points P_i curve corresponding to the domain of definition $[10]$. k times (here take $k = 3$) B spline curve control vertices by the $n+3 d_i (i = 0,1, \dots, n+2)$ controls its corresponding node vector

$$U = [u_0, u_1, \dots, u_{n+6}] \text{ according to Hartley - Judd method } [14] \text{ to}$$

calculate the knot vector

$$u_0 = u_1 = u_2 = u_3 = 0 \quad (8)$$

$$u_i - u_{i-1} = \frac{\sum_{j=i-k}^{i-1} l_j}{\sum_{s=k+1}^{n+1} \sum_{j=s-k}^{s-1} l_j} \quad (9)$$

In the equation, $i = k + 1, k + 2, \dots, n + 3$, l_j controlling each side of the polygon.

$$u_{n+3} = u_{n+4} = u_{n+5} = u_{n+6} = 1 \quad (10)$$

2) control points and weights factor solution

First, to solve the unknown control points for interpolating data points P_i ($i = 0, 1, \dots, n$) of k ($k = 3$) B-spline curve is expressed as:

$$P(u) = \sum_{j=i-3}^i d_j N_{j,3}(u) \tag{11}$$

Equation domain of $u \in [u_i, u_i + 1]$ fry $[u_3, u_n + 3]$. The node values are substituted into the curve defined within the above equations, interpolation must meet the following conditions:

$$P(u_{i+3}) = \sum_j^{i+3} d_j N_{j,3}(u_{i+3}) \tag{12}$$

For the period closed cubic B-spline curve $P_0 = P_n$, the above equation a little, then closed curve represents unity to an open curve, so there $d_0 = d_n, d_1 = d_{n+1}, d_2 = d_{n+2}$. Thus, the solution of the above-mentioned equations contains n equations to obtain all control points.

Cubic NURBS curve is defined by the $n + 3$ control points to control:

$$P(u) = \frac{\sum_{j=0}^{n+2} \omega_j N_{j,3}(u) d_j}{\sum_{j=0}^{n+2} \omega_j N_{j,3}(u)} \tag{13}$$

ω_j as a control point weight factor; $N_{j,3}(u)$ is a decision node vector U 3 specification B-spline basis functions, d_j control vertices. Weighting factor is the very nature NURBS curve, both in power factor ω_j 1:00, NURBS curve degenerated into a cubic B-spline curves.

2.2 curve fitting

By Ding pieces of the complexity of the case two fracture surfaces exact match is usually small, that is, their profile curve rarely identical, in most cases, two fragments of broken lines only partially overlap, that is only part of the outline of fracture match, so the match-inch based on the contour curve, you need to contour segmentation, two fault line as long as there is a segment contour overlap, they may have overlapped partial fragment.

It became a segmentation contour curve shows feature string, you can use string matching the idea to feature string of horses

Hotels, thereby calculating the similarity of the two curves.

String matching algorithms include two, one is to find the longest common substring of two strings, require the original substring is a contiguous string of characters, which can not insert or delete characters, another algorithm is to find two longest common subsequence of string, a string of sub-sequence refers substring delete the original part of the character string obtained, showing sub-sequence of consecutive characters in the original string is not necessarily continuous. For example, define two strings $X = "1232432"$ and $Y = "243123"$, then their longest common strings have two "123" and "243", and also has the longest common subsequence of two "2432" and "2323"

If finding the longest common substring matching the contour of the curve, the two curves corresponds to looking in the same period of the curve. The use of the longest common subsequence contour curve matching, looking for the equivalent of two curves in the interval of a similar period of the curve, of course, the separation distance can not be made too large, because of the complexity, as well as discrete sampling error of the fracture surface contour curves, exact match interval may be very small, so we have to match two contour curves by finding the longest common subsequence.

The contour segmentation curve l_1 provided a total of m vertices, denoted by $l_1 = \{p_1, p_2, \dots, p_m\}$, calculate the curvature and torsion of each vertex of the string get wherein: $l_1 = \{p_{(\kappa,\tau)}^1, p_{(\kappa,\tau)}^2, \dots, p_{(\kappa,\tau)}^m\}$, piecewise curve l_2 there are n vertices, denoted as $l_2 = \{p_1, p_2, \dots, p_m\}$ features to curvature and torsion indicated string as: $l_2 = \{p_{(\kappa,\tau)}^1, p_{(\kappa,\tau)}^2, \dots, p_{(\kappa,\tau)}^m\}$, the vertex p_1 curve l_1 and l_2 on a similar distance on a vertex p_1 is defined as follows:

$$d = \sqrt{(\kappa_{1i}^x - \kappa_{1i}^y)^2 + (\tau_{1i}^x - \tau_{1i}^y)^2} \tag{14}$$

Theoretically, if l_1 and l_2 two curves match the corresponding point on which all similar distance should be 0, that is, the two curves are equal. But in practice, because the data collection and calculation errors, matching the actual profile curve is not exactly the same, so the need to set a threshold error ϵ , so long as a similar distance between two points is less than the error threshold ϵ , considered two points Similarity.

Seek sequence $X = (x_1, x_2, \dots, x_m)$ and $Y = \{y_1, y_2, \dots, y_n\}$ basic methods longest common subsequence is calculated^[15] according to the following recursive method:

$$L[i][j] = \begin{cases} 0 & i = 0, j = 0 \\ L[i - 1][j - 1] + 1 & i, j > 0; x_j = y_j \\ \text{Max}\{L[i][j - 1], L[i - 1][j]\} & i, j > 0; x_j \neq y_j \end{cases} \tag{15}$$

Wherein the length $L[i][j]$ for the X and Y of the longest common subsequence defined X, Y similarity is:

$$\xi = \frac{L[i][j]}{\min\{m, n\}}$$

Vaguely defined as the maximum distance factor indices i and j allowed, that is, two matching characters allowed number of other characters, through the fuzzy matching factor can control the accuracy of the two feature strings, when the fuzzy factor is equal to 0, is the most long common substring.

The time complexity of the method is $O(mn)$, as used herein, Li Xin and other methods^[16] calculated the similarity of sub-curves l_1 and l_2 , fuzzy factor size is set to 2, when the similarity is greater than a given threshold, think both similar. The algorithm time complexity is $O(p(m-p))$, where p is the length of the longest common substring.

2.3 Examples of verification and analysis

The method according to the paper a lot of point cloud fragments were verified the following results:

Table 1 segment profile curve matching results

curve 1	feature points	curve 2	feature points	the length of the LCS	similarity ξ
C_{11}	149	C_{21}	147	121	0.823
C_{11}	149	C_{22}	120	39	0.325
C_{11}	149	C_{23}	109	19	0.174
C_{11}	149	C_{24}	97	16	0.164

II. summary

In order to achieve some pieces of important research value of restructuring, this paper aiming at the shortcomings of the current existing algorithm proposed a new spatial 3 d irregular fragments stitching method. First uses the method to establish point cloud based on kd - tree search space the topology relationship, for k neighborhood search quickly, further realize the boundary extraction of point cloud fragments; secondly using nurbs curve interpolation method to deal with boundary feature points, complete boundary fit at the same time the curve interpolation points of curvature and torsion; finally based on the calculation of curvature and torsion, according to the longest common subsequence complete curve matching, and then complete the point cloud fragments of stitching. Through examples show that the proposed algorithm good robustness, high matching precision.

References

- [1]. Besl P, McKay N D . A method for registration of 3D shapes [J] . IEEE Transactions on Pattern Analysis and Machine Intelligence, 1992, 14(2) : 239-256;
- [2]. Ucoluk G, Toroslui H. Automatic reconstruction of broken 3-D surface objects. Computers and Graphics, 1999, 23(4);
- [3]. Wolfson, H. J. On curve matching[J], IEEE Trans. Pattern Anal. Machine Intell. 12(5) (1990), 483-497;
- [4]. Steven E, Pierre B. Algorithm Reconstruction of Broken Fragments[J];
- [5]. Shin H, Doumas C, Funkhouser T. et al. Analyzing fracture patterns in Theran wallpaintings. Proceedings of the 11th International Conference on Virtual Reality, Archaeology and Cultural Heritage Pages 71-78;
- [6]. G. Oxholm, and K. Nishino. Reassembling Thin Artifacts of Unknown Geometry. In Vast 2011. The 12th VAST International Symposium on Virtual Reality, Archaeology and Cultural Heritage Pages 49-56;
- [7]. DING Xian Feng WU Hong and ZHANG Hong Jiang. Review on Shape Matching[J]. Acta Automatica Sinica, 2001, 27(5):678-694;
- [8]. PAN Rong Jiang , MENG Xiang Xu, and TU ChangHe. Fragment Re-Assembly Based on LCS Matching[J]. Chinese Journal of Computers, 2005, 28(3):350-356;
- [9]. LU Ke, GENG Guo hua, ZHOU Ming quan. Matching of 3D Curve Based on the Hash Method[J]. Acta Electronica Sinica, 2003, 31(2):294-296;
- [10]. Wang Jian, Zhou Lai shui ,Zhang Li yan. A Novel Algorithm on Space Curve Matching[J]. China Mechanical Engineering, 2006, 17(16):1744-1747;
- [11]. Zhou Shu chneg. On Mosaicing of 3D Complex Shape and Restoration of fragmented Objects[D]. Xi'an Northwest University, 2008;
- [12]. ZHU Yan juan, ZHOU Lai shui, ZHANG Li yan Algorithm for Three-Dimensional Fragments Reassembly[J]. Journal of Image and Graphics, 1(12):164-170;
- [13]. Chen Yiren, Wang Yibin, Peng and Zhang Jie. Improved algorithm for extraction of boundary characteristic point from scattered point cloud[J]. Computer Engineering and Applications, 2012, 48(23):177-179;
- [14]. Hartley D J, Judd C J. Parametrization of Bezier-type B-spline curves and surfaces[J]. Computer Aided Design, 1978, 10(1):130-134;
- [15]. Daniel S. Hirschberg. Algorithms for the Longest Common Subsequence Problem[J]. J. ACM, 1977, 24(4):664-675;
- [16]. Li Xin and Shu Feng di. Improved Longest Common Subsequence Problem Fast Algorithm[J]. Application Research of Computers, 2000, 2:28-30;